AgBioForum, 12(1): 8-13.

The *Santaniello* Theorem of Irreversible Benefits

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Irreversible benefits favor an earlier introduction of GM crops versus a later one. A non-trivial question is if they also weigh more than reversible benefits similar to irreversible costs but in the opposite direction.

In this contribution, I will show that indeed irreversible benefits do weigh more than reversible ones and indeed result in an irreversibility effect, albeit a positive one. The problem can be summarized by the following theorem:

> *"Irreversible benefits justify the immediate introduction of transgenic crops, even if future uncertainty about reversible benefits include negative benefits and traditional cost-benefit analysis, and treating all benefits and costs as reversible would reject the introduction."*

I call this theorem—in honor of Vittorio Santaniello—the "The *Santaniello* Theorem of Irreversible Benefits."

Key words: GMOs, irreversible benefits, uncertainty.

The International Consortium on Agricultural Biotechnology Research (ICABR) has held annual meetings since 1997. Vittorio Santaniello was one of the founding fathers and main organizers of the conference (Scandizzo, Zilberman, & Pray, 2009). He has been a strong supporter of agricultural biotechnologies, but was also aware of and concerned about the social and political issues surrounding the technology (Santaniello, 2005). He particularly emphasized the irreversible benefits that the technology provides in debates with people concerned about the irreversible costs of the technology.

Irreversible costs and their relevance for decision making, in general, is by now well known within the economic literature. Arrow and Fisher (1974) are the first authors to explicitly mention that irreversible costs matter *differently* than reversible costs for decision-making, and they introduced the concept of quasi-option value. Henry (1974) introduced the irreversibility effect. The financial economics literature provided the foundations for the real-option value theory by presenting an approach that allows one to derive the "fair price" for a call option (Black & Scholes, 1973; Merton, 1973).[1] The first application of the concept of valuing real investments using financial option models can be traced back to Myers (1977). While there are some subtle differences between the three concepts, the overall result—the possibility of postponing a decision, including irreversible costs, has an extra value that needs to be considered—holds for all three concepts. This extra value is present in all three approaches as, over time, additional information will become available and can be used by the decision-makers to update the expected benefits and costs and allows reconsideration of previous decisions. It is a form of Bayesian learning with explicit consideration of time.

The concepts of quasi-option values, irreversibility effects, and real-option values have been applied to a number of problems. Merton (1998), Trigeorgis (1995), and Dixit and Pindyck (1994) provide an overview of the methods and their applications. Applications to issues surrounding transgenic crops include Beckmann Soregaroli, and Wesseler (2006), Demont, Wesseler, and Tollens (2004, 2005), Knudson and Scandizzo (2000, 2001, 2002, 2006), Morel, Farrow, Wu, and Casman (2003), Soregaroli and Wesseler (2005), Weaver and Wesseler (2004, 2006), and Wesseler, Scatasta, and Nillesen (2007). Most of these applications have been presented first at one of the ICABR meetings.

1. *An investment can be seen similar to a call option, where the holder of the call has the right, but not the obligation, to exercise the call. If the call is exercised, the holder receives a stock and the investor has the right, but not the obligation, to invest and then receives the benefit stream generated against the payment of the investment. As it is not always optimal from an economic point of view to exercise a call option immediately—even if the call option is "in the money"—it is not optimal to invest immediately, even if the expected benefit stream exceeds the investment costs.*

The "*Santaniello* Theorem of Irreversible Benefits"

Vittorio Santaniello pointed out early on that not only should irreversible *costs* of the GM crop technology be considered, but irreversible *benefits* as well. This argument is less trivial than it seems for two reasons. First, obvious to many economists, irreversible benefits favor an earlier introduction of GM crops versus a later one, but this has been largely neglected within empirical studies on consumer attitudes towards GMOs. Second, and somewhat less obvious, irreversible benefits weigh more than reversible benefits in a similar fashion to irreversible costs—but in the opposite direction—and by this, introduce an asymmetry similar to the one of irreversible costs.

The importance of Vittorio Santaniello's comment can be stated in the following way:

> *"Irreversible benefits justify the immediate introduction of transgenic crops, even if future uncertainty about reversible benefits include negative benefits and traditional cost-benefit analysis, and treating all benefits and costs as reversible would reject the introduction."*

In honor of Vittorio Santaniello for his contribution to the economics and policy of agricultural biotechnology I call this the "*Santaniello* Theorem of Irreversible Benefits."

The proof of the theorem can be found in the Appendix.

Interpretation of the Theorem

A numerical example may help to appreciate the implications of the theorem. Choosing a discount rate r of 10%, NB_0 = 100, NB_1^h = 10, NB_1^l = -200, and q = 1-q = 0.5, we solve for B by following Equation 2 in the Appendix:

$$B > -100 - 0.5 \cdot \frac{20}{0.1} + 0.5 \cdot \frac{200}{0.1} > 800 \cdot$$

Following Equation 4 in the Appendix results in

$$B > -100 - 0.5 \frac{20}{0.1} > -200.$$

As the numerical example illustrates, there is a difference in necessary irreversible benefits of 1,000 units, depending on the valuation approach being used. Applying the "*Santaniello* Theorem of Irreversible Benefits," the mere presence of positive irreversible benefits would in this case justify an immediate introduction, while following the standard cost-benefit analysis, the irreversible benefits have to be at least 800 units. As can be easily seen, even negative annual incremental reversible benefits can be tolerated in the presence of irreversible benefits.

The difference in the results with and without considering the irreversible benefits effect can be explained in the different treatment of future information. In the first case, the standard cost-benefit analysis, future negative reversible net-benefits, ($NB_1^l < 0$), still enter the valuation. In the second case, the arrival of future information is considered and in the case ($NB_1^l < 0$), GM crops will be dis-adopted and enter the valuation with zero value.

While in the first case a value of $B > 800$ would support immediate introduction and support the argument for subsidizing the technology, in the second case—considering the irreversible benefit effect for the same amount of irreversible benefits—a much higher subsidy can be justified.[2]

The simple model presented in the Appendix is sufficient for proving the irreversible benefit effects, but a number of modifications are possible. These include considering irreversible costs, uncertainty about irreversible benefits and costs, irreversible benefits and costs of changing from adoption to dis-adoption of the GM crop, decrease of incremental reversible benefits, and more (e.g., Demont et al., 2005; Hennessy & Moschini, 2006).

Evidence and Implications of Irreversible Benefits

The empirical evidence for irreversible benefits in the European Union is weak. The studies for Europe only indicate small amounts of irreversible benefits, which can be mainly explained by the low use of insecticides to control the European Corn Borer and the relatively low use of herbicides in sugar beets and corn, as well as a ban of a number of harmful pesticides. Demont et al. (2004) calculated irreversible benefits for herbicide-tol-

2. *A cautious note is warranted for not getting misunderstood about the subsidy argument. The subsidy argument relates to supporting the introduction of GM crops through providing the appropriate infrastructure, such as a seed distribution system. At farm-level the technology has to pay to provide sufficient incentives for farmers using the technology.*

erant sugar beets to be about 1.60€ per hectare per year, while Wesseler et al. (2007) calculated irreversible benefits of about 0.81-1.08€ per hectare for Bt corn and 1.69-2.62€ per hectare for herbicide-tolerant crops in selected EU member states.

Vittorio Santaniello was more concerned about the irreversible benefits the introduction of GM crops will generate for developing countries, in particular by the positive effect on malnourishment and farmers' health (Santaniello, 2002, 2005). Malnourishment of young children for more than two years can result in stunted growth, negatively affecting future mental capabilities. This effect is irreversible and can even be passed down to the next generation. Reducing malnourishment can amount to a considerable irreversible benefit effect, acknowledging that by 2015 at least 400 million—and more likely 600 million—people in the world will be undernourished (FAO, 2004). The "*Santaniello* Theorem of Irreversible Benefits" indicates that perhaps much more can be gained by the introduction of GM crops in developing countries than reported by most current studies.

Another example is the control of "black Sigatoka" in bananas in places where they are a staple crop, such as Uganda. Concerns about the irreversible costs of introducing GM bananas cost the economy of Uganda anywhere from $180-365 million USD per year (Kikulwe, Wesseler, & Falck-Zepeda, 2008). The results presented by Kikulwe, Birol, Falck-Zepeda, and Wesseler (Forthcoming) show the delayed introduction in particular harms less-wealthy households in rural areas, as they express the highest willingness-to-pay for the technology.

Research on HIV/AIDS in Africa shows the number and quantity of crops grown in the home garden increases among HIV/AIDS-affected households (Gebreselassie, 2009). This opens the possibility for HIV/AIDS mitigation through improved nutritional value via biofortification of home-garden crops. Irreversible health benefits can be gained by Bt corn with lower levels of mycotoxins (Wu, 2006). The research on pesticide use in Bt cotton (Huang, Hu, Rozelle, Qiao, & Pray, 2002; Kuosmanen, Pemsl, & Wesseler, 2006; Pray, Ma, Huang, & Qiao, 2001) and insect-resistant rice in China (Huang, Hu, Rozelle, & Pray, 2005, 2008) shows a huge decrease in pesticide use. The pesticide use among Bt cotton farmers decreased by about 58%, as reported by Huang et al. (2002), and is expected to decrease among rice farmers by about 80%, as reported by Huang et al. (2005).

While the assessment of productivity and health effects of GM crops is more complex than illustrated by the numbers being presented (Scatasta & Wesseler, 2004; Waibel, Zadoks, & Fleischer, 2003), the positive irreversible health effects can hardly be denied.

Skepticism Towards Considering Irreversibilities

Many colleagues have been skeptical about using a real-option approach for analyzing the irreversible benefits and costs of GM crops. The standard criticisms are that "there are no irreversible costs," or that the approach is "complicated" and "uses many assumptions."

The "there are no irreversible costs" argument misses the point that more than half of the world's population shows reservations about the technology because of subjectively perceived irreversibilities. Ignoring those concerns does not help to increase trust in the economic analysis of costs and benefits of the technology.

In particular, in the European Union concerns about irreversible environmental effects of GM crops have been put forward as an argument for postponing the introduction (Commission of the European Communities, 1999). Actually, if there were no concerns about irreversible effects, there would be no argument against immediate introduction. This holds even under uncertainty, as benefits and costs are supposed to be reversible, and if the future turns out to be not as favorable as expected, growing of the GM crop could be stopped without any additional costs thereafter. As Paarlberg (2008), in his seminal contribution, has shown, those concerns are also important for decision makers in Africa.

Research explicitly considering potential irreversible costs (Demont et al., 2004; Scatasta, Wesseler, & Demont, 2006; Wesseler et al., 2007) actually casts doubts about irreversible costs being sufficiently large to postpone immediate introduction of herbicide-tolerant sugar beets, herbicide-tolerant corn, and Bt corn in the EU. The result of the case study from Portugal (Skevas, Fevereiro, & Wesseler, 2009) actually indicates that the reversible incremental benefits for Portugal are even larger than predicted in Wesseler et al. (2007).

The "complicated" argument reflects a misunderstanding about the approach being used. The different specifications of real-option models almost all try to investigate the value of a technology under uncertainty. The irreversibility effect often enters the analysis very much in a standard fashion by calculating the value of a call option with an uncertain underlying asset, the GM crop. The valuation of the GM crop is complicated and by this, so is the real-option value. But the value of the GM crops will be needed whether or not the real-option value or a different valuation approach will be used.

The same holds true for the widely shared view that the real-option approach "uses many assumptions." This view misses the point that having made one assumption is not having made a different one. By this, rejecting an approach by the number of assumptions is an empty argument. The more relevant question is whether the assumptions being made are reasonable. Space does not allow a discussion of this in detail, and I refer the interested reader to the book by Shreve (2005), which discusses all the details of the approach. At least, the assumptions and the approach in general seem to be convincing to many economists. The "founding fathers" have been awarded with the Noble prize in economics for the call-option pricing formula (Robert C. Merton and Myron S. Scholes in 1997) and for pricing assets under uncertainty (Harry M. Markowitz, Merton H. Miller, and William F. Sharpe in 1990), as well as the AERE (Association of Environmental and Resource Economists) Publications of Enduring Quality Award for decision-making under uncertainty and irreversibility (Kenneth Arrow and Anthony C. Fisher in 1995).

Conclusion

Vittorio Santaniello has been one of the few colleagues I met who immediately understood the relevance and implications of irreversible benefits and costs within the debate about the economics and politics of GM crops. By stressing the irreversible benefits, Vittorio Santaniello has always reminded us—and in particular those concerned about the technology—those irreversible benefits and costs are the two sides of the same coin.

The ICABR meetings have always been a place where different scholars have presented work including irreversibilities, including, just to name a few, Volker Beckmann, Matty Demont, Joze Falck-Zepeda, Richard Gray, Enoch Kikulwe, Odin Knutsen, Pasquale Scandizzo, Sara Scatasta, Claudio Soregaroli, Robert D. Weaver, David Zilberman, and myself. The presentations always have resulted in a lively debate. By this, Vittorio Santaniello has contributed another irreversible benefit.

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Appendix

Proof: For proving the theorem, consider a two-period model with t_0 indicating the present and $t_{1,\ldots,\infty}$ indicating the future, one and more years from now. The annual incremental reversible net benefits of introducing a new GM crop will be denoted by NB_t. NB_0 is known, but $NB_{t>0}$ is uncertain. For simplifying the proof, uncertainty will be fully resolved at an infinitesimal time step before t_1. At t_1, NB_1 can reach two states of nature, either NB_0 has changed with probability q to $NB_1{}^h$ or with probability (1-q) to $NB_1{}^l$. For avoiding triviality, assume $NB_1{}^l < 0$. The appropriate discount rate will be denoted by r. The objective of the decision-maker is to maximize the total benefit of introducing the GM crop.

The strategy is to first develop the decision criterion ignoring the irreversibility effect of irreversible benefits and then to compare the result with the one including the effect.

The total incremental reversible net benefits of introducing the new GM crop at farm level will be

$$NB_0 + q\frac{NB_1^h}{r} + (1-q)\frac{NB_1^l}{r},$$

and by considering external constant irreversible benefits B, immediate introduction would be justified if

$$B + NB_0 + q\frac{NB_1^h}{r} + (1-q)\frac{NB_1^l}{r} > 0, \quad (1)$$

or the decision will be choosing

$$\max\left\{B + NB_0 + q\frac{NB_1^h}{r} + (1-q)\frac{NB_1^l}{r}, 0\right\}. \quad (2)$$

Here, we can already observe that irreversible benefits do have a positive affect and increase the likelihood of introduction in comparison to the situation where they are not present. But what is also obvious is that they have the same affect as an increase in NB_0.

The value of introducing the new GM crop one year from now, considering using additional information—assuming $\left(B + NB_1^l + \frac{NB_1^l}{r}\right) < 0$—again to avoid triviality, and considering that growing the GM crop is reversible:

$$\max\left\{q\left(B + NB_1^h + \frac{NB_1^h}{r}\right), 0\right\} \text{ at } t=1. \quad (3)$$

From this directly follows the value of introducing the GM crop immediately, considering the arrival of future information with $NB_0 > 0$:

$$\max\left\{B + NB_0 + q\frac{NB_1^h}{r}, 0\right\}. \quad (4)$$

Comparing Equation 2 and Equation 4, we immediately observe for $NB_1{}^l < 0$

$$q\frac{NB_1^h}{r} > q\frac{NB_1^h}{r} + (1-q)\frac{NB_1^l}{r} \blacksquare \quad (5)$$

The "*Santaniello* Theorem of Irreversible Benefits" implies an irreversibility effect in a two-states-two-times model of (1-q) ($NB_1{}^l$ / r). By this factor, necessary irreversible benefits can be smaller in comparison to an approach that does not consider the arrival of future information. Following standard dynamic optimization procedures, moving from discrete time to continuous time and assuming future incremental reversible benefits following a binomial distribution converging to continuous state under infinity provides the following result for irreversible benefits:

$$B^* = \frac{NB_0/\rho}{\frac{\beta_1}{\beta 1 - 1}}, \quad \text{with } \beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2}\sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \quad (6)$$

assuming appropriate boundary conditions, where β_1 is the solution of a second order homogeneous equation, δ the convenience yield, and σ the variance rate of a geometric Brownian motion. For the details, see, e.g., Dixit and Pindyck (1994) or McDonald and Siegel (1986).